

Indian Statistical Institute, Bangalore

M. Math. Second Year

Second Semester - Simple Groups and Geometry

Final Exam

Duration: 3 hours

Date : May 05, 2015

Answer all the questions. Your answer should be clear and complete. Max Marks: 100

1. (a) Define a nondegenerate symplectic bilinear form on a vector space. Show that if a finite dimensional vector space admits a nondegenerate symplectic bilinear form, then the dimension of the vector space is even. [4+8]
(b) Define an ordered symplectic basis of a finite dimensional vector space over \mathbb{F}_q , equipped with a nondegenerate symplectic bilinear form. Compute the number of ordered symplectic bases of V . [4+10]
2. Define the concept of imprimitivity of a transitive permutation group. Show that the action of the symplectic group on the set of points of the corresponding projective space is imprimitive. [6+8]
3. Define an (s, t) -generalized hexagon. Find the number of points at a maximum distance from a given point. [6+9]
4. Define a transvection of a vector space V . Show that the transvections form a conjugacy class in $SL(V)$ if dimension of V is atleast three. [3+7]
5. (a) Find the number of three dimensional subspaces of \mathbb{F}_q^6 .
(b) Show that $X_0^2 + X_1X_2 = 0$ defines a $q + 1$ - arc in the projective plane over \mathbb{F}_q . [8+12]
6. (a) Show that $Aut(S_3)$ is isomorphic to S_3 .
(b) Show that S_6 contains a subgroup of order 120 which is transitive on the six symbols. [5+10]

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